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ABSTRACT

At issue in this study was the extent to which large numbers of classroom teachers were able to implement research-based materials with a minimum of inservice education. Also at issue was whether students of these teachers were able to develop the rich mental images for fractions similar to the ones students from previous Rational Number Project (RNP) studies developed in smaller experimental settings. An analysis of student interviews (n=20) in fourth grade in a suburban school district south of St. Paul and Minneapolis, Minnesota demonstrated that RNP students did in fact develop rich mental images for fractions similar to students in previous studies. As expected, the nature of RNP students' thinking about rational numbers was far richer and indicated a more conceptually oriented framework than that of students who used textbook curriculum. (Author/MKR)

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# Facilitating Children's Development of Rational Number Knowledge

**Kathleen Cramer and Thomas Post**

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# FACILITATING CHILDREN'S DEVELOPMENT OF RATIONAL NUMBER KNOWLEDGE

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At issue in this study was the extent to which large numbers of classroom teachers were able to implement research-based materials with a minimum of inservice education and whether students of these teachers were able to develop the rich mental images for fractions similar to the ones students from previous Rational Number Project (RNP) studies developed in smaller experimental settings. An analysis of student interviews demonstrated that RNP students did in fact develop rich mental images for fractions similar to students in previous studies. As expected the nature of RNP students' thinking about rational number was far richer than students who used textbook curriculum and indicated a more conceptually oriented framework.

## Background

Since 1980, the RNP has reported on many investigations into the teaching and learning of fractions among fourth and fifth graders ( Bezuk & Cramer, 1989; Post, Wachsmuth, Lesh & Behr, 1985). The curriculum used in the study reported here emanated from this earlier research. The RNP curriculum used in earlier studies reflected the following beliefs: (a) Children learn best through active involvement with multiple concrete models, (b) physical aids are just one component in the acquisition of concepts-verbal, pictorial, symbolic and realistic representations also are important, (c) children should have opportunities to talk together and with their teacher about mathematical ideas, and (d) curriculum must focus on the development of conceptual knowledge prior to formal work with symbols and algorithms.

The curriculum developed the following topics: (a) part-whole model for fractions, (b) concept of unit, (c) order ideas , (d) equivalence concepts and (e) addition and subtraction of fractions at the connect level. It de-emphasized standard paper-pencil procedures for ordering fractions, finding fraction equivalencies and symbolic procedures for operating on fractions. Instead it emphasized the development of a quantitative sense of fraction. To think quantitatively about fractions, students should know something about the relative size of fractions and be able to estimate reasonable answers when fractions are operated on.

The fraction curriculum used in earlier investigations was revised and extended. The goal for this revision was to reorganize lessons from the 30-week teaching experiment into two levels of teaching materials that could be used easily by classroom teachers with fourth and fifth grade students. This study used Level 1 materials (23 lessons) with all students regardless of grade level. Some lessons lasted more than one day.

## Treatments

### RNP Curriculum

The curriculum was written to reflect cognitive psychological principles as suggested by Piaget (1960), Bruner (1966), and Dienes, (1967). Lesh (1979) elaborated on their ideas and produced a model which suggests that learning is enhanced when children have opportunities to explore mathematical ideas from multiple perspectives - manipulatives, pictures, written symbols, verbal symbols and real life contexts. The model also suggests that it is the translations within and between modes of representation that make ideas meaningful for children. The RNP curriculum reflects this theoretical model. The manipulatives used in the lessons included fraction circles, chips, and paper folding.

An important part of each lesson is the "Notes to the Teacher" section. Here insights into student thinking captured from the initial RNP teaching experiments are communicated to teachers. The notes share examples of student misunderstandings and anecdotes of student thinking from earlier RNP projects. These notes to the teachers also clarify methods for using manipulative materials to model fraction ideas.

### Textbook Curriculum

The majority of 33 textbook classrooms used the 1989 edition of *Addison-Wesley Mathematics* (AW). Six of the textbook classrooms piloted the Harcourt Brace Jovanovich (HBJ) 1992 series, *Mathematics Plus*.

The textbook teachers were encouraged to use the resources suggested in the teacher's guides. Fraction bars and pictures of fraction bars were the models suggested by Addison-Wesley textbook series. The HBJ series suggested a larger variety of manipulative materials. These included counters, paper folding, fraction circles and fraction bars made from paper strips. In each case, though, concrete models played only a cursory role in the development of fraction ideas; the primary goal was to develop student competence at the symbolic level. In the RNP lessons translations including extensive use of manipulative materials were the central focus. Symbols were used to record students' observations, discussions and actions with manipulatives.

## Procedures

### Treatment Assignments

In a suburban school district south of the Twin Cities all 200 fourth and fifth grade teachers were contacted in the fall of 1993 to assess their interest in participating in this study. Sixty-six teachers from 17 schools chose to participate. Teachers were assigned to treatment conditions ( RNP or Textbook) by grade level. There were 38 fourth grade classrooms; 19 RNP and 19 Textbook classrooms. There were 28 fifth grade classrooms; 14 RNP and 14 Textbook classrooms.

Timeline

The study began with the first of two, two-hour teacher inservices. This inservice was divided into two parts. For the first hour, all teachers heard a presentation that dealt with the following topics: (a) history of the RNP, (b) structure of the study, and (c) research on student learning of fractions. The second hour, teachers broke up into two groups by treatment and reviewed the goals and objectives of their respective treatments conditions.

Instruction was to last a minimum of 28 days and a maximum of 30 days; each class period was for 50 minutes. A second, two-hour inservice session was held half way through instruction. In the first hour all teachers participated in a discussion on assessing fraction learning. During the second hour the RNP teachers worked through activities with manipulatives modeling fraction addition and subtraction. The textbook group considered several fraction enrichment activities.

Interviews

Twenty, fourth graders were interviewed by project staff, each from a different classroom. Ten were selected from the RNP group and 10 from the textbook group. Each student was interviewed three times. Interview topics included concepts, order, equivalence, concept of unit and fraction operations. Each classroom teacher also randomly selected three students from his/her class and interviewed them once at the end of the study. This interview included eight questions in those same five areas.

Questions:

The following questions were of interest:

1. Is the RNP curriculum written and organized so teachers can use it effectively with limited inservice opportunities?

2. Do students taught by classroom teachers using the RNP curriculum develop similar understandings for fractions as compared to students in original RNP teaching experiments taught by project investigators?

3. What differences occur in student achievement and student thinking between students using a conceptually-oriented RNP curriculum and students using district-adopted textbooks?

To investigate these three questions, the RNP personnel relied on several different data sources. Post and retention written test data from some 1600 fourth and fifth grade students provided the foundation from which answers to each of the above questions were generated. Interviews with RNP students were to determine whether student thinking documented during the teaching experiments could be replicated on a large scale with students taught by classroom teachers using RNP curriculum and to provide anecdotal data depicting treatment differences in student thinking.

The remainder of this paper addresses interview results related to differences in student thinking between RNP and Textbook students. Table 1 reflects data from the final interview given to 17 fourth graders by project staff. [Note: Of the 10 RNP student originally selected to be interviewed, two were tracked into a low math group and did not finish the lessons. These two students were not given the final interview. One Text student did not take the final interview. Data reflecting numbers less than eight RNP students and nine Text students interviewed represent missing data]. Table 2 reflects data from interviews with fourth graders given by classroom teachers. Each table reports student responses to an estimation question. A student response which relied on mental images for the fractions to determine their relative size was categorized as a conceptual response. This was in contrast to a student response that solely relied on symbolic procedures to estimate. Here little thought as to the relative sizes of the fractions was considered. Students determined the exact answer and then estimated from that exact answer. Student responses to the problem in Table 1 are organized below that table to exemplify their correct and incorrect answers. Student responses similar to those reported here also were found in interviews given by classroom teachers. Limited space prohibits a detailed list of examples.

**Table 1: Final interview given by project staff**

Marty was making two types of cookies. He used 3/12 cup of flour for one recipe and 2/3 cup for the other. How much flour did he use altogether? *Without working out the exact answer, give me an estimate that is reasonable. ( If needed ask: Is it >1/2 or <1/2? >1 or <1?)*

| RNP (8 students) | | | | | TEXT (9 students) | | | | |
|---|---|---|---|---|---|---|---|---|---|
| correct concept | correct procedure | correct no explan | incorrect | missing data | correct concept | correct procedure | correct no explan | incorrect | missing data |
| 6 | | | 1 | 1 | | 2 | 2 | 4 | 1 |

**RNP Correct Responses:**

**K.E.:** It would be more than 1/2. It would be less than a whole. If you had 2-thirds, that's more than half and then you put 3-twelfths to add to it, it would not be a whole.

When asked how she knew it wasn't going to be a whole she said: 3-twelfths isn't very big so you'd add a little more.

**J.S.:** About 1 whole. The 3-twelfths - I think 3 of these could fit in the missing spot.

**M.G.:** Greater than 1/2, less than one. 2/3 is almost a whole. 4-twelfths plus 2-thirds equals one; so 3/12 plus 2-thirds is not quite one. [All done mentally].

**B.S.:** About one. 1/3 is bigger than 1/12. Then 3/12 wouldn't equal 1/3. And you need 2 more thirds to equal a whole.

**A.R.:** Greater than 1/2. It takes 3 reds to cover one blue [fourths] so it probably takes 4 reds to cover a brown [thirds]. So there's only 2 of 3 [browns]. There's a gap when you fill with 3 reds.

**K.B.:** I know it's greater than 1/2 because 2/3 is greater than 1/2. Close to one, a little less. Because I just think that.

### RNP Incorrect Responses

**L.C.:** About one. 3-twelfths equals 1-third; 2-thirds plus 1-third equals one.

### TEXT Correct Responses

**A.B.:** Greater than 1/2 and less than one. I know how many times this could go into 12 is four and you go four to get the denominator. And it was four times three, you take three times four equal twelve and then two times four equals eight and then you get 8-twelfths. Then you go 8-twelfths plus 3-twelfths equals 11-twelfths and then its more than 1/2 and less than one.

**L.B.:** Greater than 1/2; less than one. Couldn't explain why.

**E.S.:** Greater than 1/2; less than one. I am just guessing.

**M.C.:** About 1 and a little over. You round this off to twelfths [points to 2/3]. Quadruple that [2] to 8 , (add to 3/12), that's approximately one whole. [He estimates after mentally arriving at exact answer]

### TEXT Incorrect Responses

**K.H.:** Less than 1/2. Unable to explain reasoning

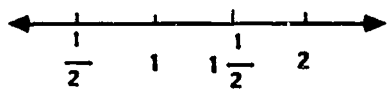
**N.D.:** I don't know. Can't guess. [Wanted to use paper and pencil]

**K.A.:** More than one. I don't know. It just seems high.

**B.A.:** He ate about 1/2. I subtract it. I can't do it in my head.

### Table 2: Interviews given by classroom teachers

Tell me about where 11/12 - 4/6 would be on this number line:




| correct concept | correct proced | correct expl unclear | correct no expl | correct pict/manip used | incor concept | incor proced | incor expl unclear | incor no expl | miss data |
|---|---|---|---|---|---|---|---|---|---|
| | | | | RNP Group (53 students) | | | | | |
| 20(41%)* | 5(10%) | 2(4%) | 2(4%) | 8(16%) | 7(14%) | 2(4%) | | 3(6%) | 4** |
| | | | | Text Group (57 students) | | | | | |
| 2(6%) | 14(41%) | | | | 3(9%) | 6(18%) | 3(9%) | 6(18%) | 23** |

*Percentages based on available data: RNP 49 students; Text 34 students.

** Missing data for the RNP group represents students who did not complete the 23 lessons. Teachers chose not to ask students this question. Missing data for the textbook group represents teacher error. Several teachers asked the wrong question. They asked students to place the two fractions on the number line instead of the difference between the two numbers.

Discussion

Almost all the RNP students were able to provide a reasonable estimate to the addition problem given and a large percentage could estimate a subtraction problem. It should be noted that of the 23 lessons only five dealt with the arithmetic operations. These lessons developed addition and subtraction concretely and within context. Paper and pencil procedures for finding fraction equivalencies and common denominators were not taught.

Differences in students' thinking about fractions are evident. RNP students' responses relied on their mental images for fractions considered. Images described relate directly to fraction circles, the manipulative used most frequently in the lessons. Students used images to determine a fraction's relative size (2/3 > 1/2; 2/3 is almost a whole) as well as to determine simple equivalencies (3/12 = 1/4; 3 reds equals 1 blue). Textbook students' responses show most students did not use mental imagery to reason through an addition or subtraction problem. Textbook students most often relied on symbolic procedures (find exact answer and then estimate) or were unable to verbalize reasons for their estimate.

Differences in students' ability to verbalize was evident. RNP lessons emphasized student discussion of ideas and translations to and from the verbal mode of representation. The manipulatives themselves became a focal point for student discussion - students talked about their actions with manipulatives.

The initial studies conducted by the RNP have provided much information and insight into issues involving the teaching and learning of fractions. Our goal for this study was to organize a large scale implementation of a curriculum based on this previous research. Results here provide evidence that large numbers of classroom teachers can effectively implement well structured, conceptual-based curriculum which in this study resulted in student learning that was rich in conceptual understandings as contrasted to the procedural-based learning characterized by students using the textbook curriculum.

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